

Electromagnetic Field Distribution of Conducting Polygons with Wedge Cavities

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Abstract: Electromagnetic scattering from conducting polygons is studied. We will discuss the resonant characteristics of conducting cylinders with wedge cavities and clarify the electromagnetic field distribution inside the cavities using PMM.

1. Introduction

Analysis of electromagnetic scattering is important for target recognition and reduction of the RCS (Radar Cross Section) [1]. When a target has an aperture, the scattering phenomena become more complicated due to the multiple scattering and resonance in the interior region of the aperture; hence, development of a highly reliable computational technique is important. The authors have developed the PMM (Point Matching Method) taking account of the edge conditions and reported that electromagnetic scattering problems can be analyzed with high accuracy. In this paper, we will discuss the resonant characteristics of conducting cylinders with wedge cavities and clarify the electromagnetic field distribution inside the cavities using PMM.

2. Formulation

A scatterer shown in Figure 1 is the rectangular cylinder with the wedge cavity. It is assumed to be uniform along the z-axis, and the cross section is $2a \times 2b$.

We impose the symmetry due to the $x$-axis on the scatterer. Hence the incident wave for the $H$-polarized case in the cylindrical coordinate system $O(r, \theta)$ is decomposed into the two components as

$$ H_0^+(\rho) = \frac{H}{2} \left\{ \exp[jkr\cos(\theta - \phi)] + (-1)^p \exp[jkr\cos(\theta + \phi)] \right\} $$

(1)

where $k$ is the wavenumber in free space and $\phi$ is the angle of incidence. The even-phase component is represented by $p = 0$ and the odd-phase component is represented by $p = 1$. The time dependence is assumed to be $e^{j\omega t}$ and suppressed throughout the paper.

For this scattering problem, the upper half-space ($y \geq 0$) can be decomposed into the seven regions as shown in Figure 2. The electromagnetic field in each region can be expanded using the mode which satisfies the Helmholtz equation inside the local coordinate system. We can define all the separated regions and explain the electromagnetic fields as follows;

- **Region S_1**: Outside the circle $C_1$ [origin $O$, radius $\rho_a$]

  The scattered field in this region satisfies the radiation condition. Therefore it can be approximated using a finite sum of modes in the coordinates,

  $$ H_{1,S_1}^{(1)}[\rho] = \sum_{n=0}^{N} A_{n}\{H_{n}^{(1)}(kr)\cos(n\theta - \frac{\pi}{2}p)\} $$

  (2)

  where $\rho_a := (1 + \delta)\sqrt{a^2 + b^2}$, $H_{n}^{(1)}(\cdot)$ is the $n$-th order of the second kind of Hankel function, and $N$ is the truncation number.

- **Region S_2**: Inside the circle $C_2$ [origin $O$, radius $\rho_a$]

  This region encloses the open-ended. The field can be expanded using the combination of trigonometric functions. The magnetic field can be written in the local coordinate system as

  $$ H_{1,S_2}^{(1)}[\rho] = \sum_{n=0}^{M} J_n(\rho_b)\left[(1 - p)\beta_0^n[0]\cos(n\theta) + p\beta_0^n[1]\sin(n\theta)\right] $$

  (3)

  where $M$ is the truncation mode number.

- **Region S_3**: Inside the circle $C_3$ [origin $O$, radius $\rho_1$]

  To satisfy the edge condition, the magnetic field can be written in the local coordinate system as

  $$ H_{1,S_3}^{(1)}[\rho] = \sum_{n=0}^{M} B_n(\rho_c)J_{m+n}\left[(\rho_c\rho_1)^{\frac{\pi}{\alpha_1}}\right] $$

  (4)

  where $m = n (l = 2 - 5)$, $m = 2n + 1 (l = 5)$, $J_{m}$ is the $m$-th order of the Bessel function, $M$ is the truncation mode number for each separated region, and $\alpha_1 = \pi/\alpha_1$.

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Region $S$: Inside the circle $C$ [origin $O$, radius $\rho$]

This region can be expanded using the combination of the $n$-th order of the Bessel and Neumann functions as

$$H^{(1)}_n[p] = \sum B_n^{(1)}[p] J_{n}(kr) + D_n^{(1)}(kr) \cos(n(2n + p)\theta),$$

where $D_n = -J_n^{\prime}(kd)/N_n^{\prime}(kd)$, $kd$ is the distance $MP$

3. Computational Results

To verify the computational accuracy, the convergence test for varying the truncation mode number $N$ is performed. Figure 3 shows the convergence process of the RCS at $\theta = 0$ when the incident $H$-polarized wave impinges from $\phi_0 = 0^\circ$. The geometrical parameters are $ka = \pi$, $a/b = 1$, $\alpha_1 = 8^\circ$ and $\alpha_2 = 356^\circ$. In this case, we can obtain 8-digit accuracy when $N > 89$.

Figure 4 shows the monostatic RCSs for varying $ka$ when $\alpha_2 = 358^\circ$, $\alpha_2 = 356^\circ$, and $\alpha_2 = 354^\circ$, they are almost identical expect some resonant peaks. Figure 5a is a plot of the magnetic field distribution at the resonance peaks at $ka = 1.85$ for $\alpha_2 = 356^\circ$. The amplitude is almost uniform along the $ky$-axis. Therefore, the resonant peaks are not affected by the wedge angle $\alpha$. Figure 5a is a plot of the magnetic field distribution at the resonance peaks at $ka = 1.85$ for $\alpha_2 = 356^\circ$. The amplitude is almost uniform along the $ky$-axis. Therefore, the resonant peaks are not affected by the wedge angle $\alpha$. Figure 5b is a plot of the magnetic field distribution at the resonance peaks at $ka = 3.86$ for $\alpha_2 = 356^\circ$. In this peak, the resonant phenomenon is affected by both $ky$ and $kx$-axes. Therefore, the peak depends on the wedge angle $\alpha$.

4. Conclusions

In this paper, we discuss the resonant characteristics of conducting cylinders with wedge cavities and clarify electromagnetic field distribution inside the cavities.

5. References