K7-26

Development of Flight Simulator for Space Transportation System

-Construction of motion calculation system for space transportation-

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Abstract: The authors have developed a special-purpose flight simulator for a space transportation mission. The simulator consisted of a computer system for simulating the equations of motion of a space plane, and included aerodynamic data for the Space Shuttle and a visual system for projecting images onto three screens. However, the equations of motion of the vehicle employed were derived using a flat-Earth model. That is, the influence of centrifugal and Coriolis forces due to the curvature of the Earth were not taken into consideration. Therefore, in this paper, the authors attempt to rewrite the equations of motion to allow simulation of a flight from a circular orbit around the Earth to a runway on the ground through reentry into the atmosphere.

1. Introduction

The authors have developed a special-purpose flight simulator that can simulate a mission from atmospheric reentry to landing on a runway. This simulator consisted of a control system to generate the command signals to the onboard computer, a computer system for simulating the equations of motion, and a visual system to output external images and status signals to screens and instruments.

However, in the above mentioned computer system, a local coordinate system fixed on the runway was used for the inertial axes. Thus, it cannot be used to describe a long flight path from an Earth-circular orbit to a runway on the ground.

Therefore, in the present paper, the authors attempt to construct the kinematics of a vehicle based on an Earth-centered inertial coordinate system and to rewrite the equations of motion to take into account the influence of centrifugal and Coriolis forces.

2. Configuration of the flight simulator

Figure 1 shows the configuration of the flight simulator. Command signals from the control system (Fig. 1(a)) are transmitted to an onboard computer (Fig. 1(b)), and the equations of motion are calculated using aerodynamic data from the U.S. Space Shuttle and standard atmospheric conditions on the DSP board®. Based on the state variables and the parameters generated by the software, instrument indicators and external images are displayed (Fig. 1(c,d)).

In particular, in the control system, an unsymmetrical operation range of the elevons and vertical tail of the Space Shuttle is taken into consideration. Additionally, an algorithm for the reaction control system which enables the vehicle to control its position in a rarefied atmosphere at high altitude will be constructed in the future.

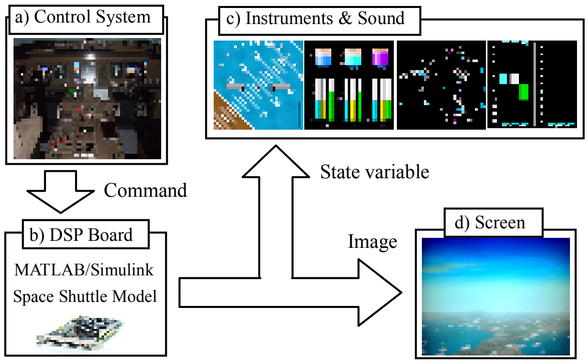

Figure1. Configuration of Flight Simulator

3. Coordinate system

In general, when describing the motion of an aircraft, an inertial coordinate system centered on the Earth's surface is used; i.e., a flat-Earth model.

However, because the flight path of a space transportation system is much longer than that of a conventional aircraft, the curvature and rotation of the earth must be considered.

Therefore, the equations of motion must be described in terms of an Earth-centered coordinate system. The effects of centrifugal and Coriolis forces can then be taken into account. The inertial coordinate system used in the present study is shown in Fig. 2 (System *I*). The kinematics of the vehicle is described with an Earth-centered rotating coordinate system (System *E*), a local flat coordinate system (System *L*), and the body axes (System *B*).

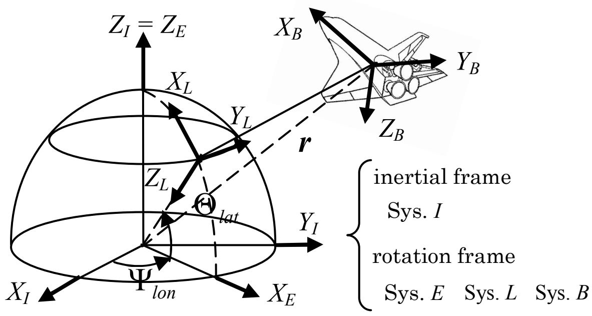

Figure2. Coordinate systems

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The relationship among these coordinate systems and their transformation matrices are given as follows:

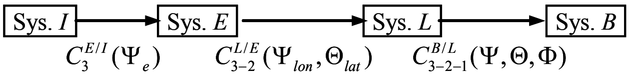

Figure3. Coordinates and Transformation Matrices

Here, Ψ_e, Ψ_{lon} and Θ_{lat} are the angle of the earth rotation, the longitude and the latitude, respectively. The body axes component (System B) of the angular velocity relative to the inertial axes (System I) is denoted by $\boldsymbol{\omega}^{B/I}$. This angular velocity $\boldsymbol{\omega}^{B/I}$ is determined as the summation of the angular velocities $\boldsymbol{\omega}^{L/I}$ and $\boldsymbol{\omega}^{B/L}$.

$$\boldsymbol{\omega}^{B/L} = \begin{bmatrix} \boldsymbol{i}_B & \boldsymbol{j}_B & \boldsymbol{k}_B \end{bmatrix} \begin{bmatrix} P^{B/L} & Q^{B/L} & R^{B/L} \end{bmatrix}^T \tag{1}$$

$$\boldsymbol{\omega}^{L/I} = \begin{bmatrix} \boldsymbol{i}_L & \boldsymbol{j}_L & \boldsymbol{k}_L \end{bmatrix} \begin{bmatrix} P^{L/I} & Q^{L/I} & R^{L/I} \end{bmatrix}^T \tag{2}$$

$$\boldsymbol{\omega}^{B/I} = \boldsymbol{\omega}^{B/L} + \boldsymbol{\omega}^{L/I} \tag{3}$$

Here $\boldsymbol{\omega}^{Y/X}$ indicates the angular velocity of System Y relative to System X. Besides, rewriting Eq. (3) in terms of the Euler angles $\dot{\Phi}, \dot{\Theta}, \dot{\Psi}$ relative to System L gives the following equation.

$$\begin{bmatrix} \dot{\Phi} \\ \dot{\Theta} \\ \dot{\Psi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\Theta \\ 0 & \cos\Phi & \sin\Phi\cos\Theta \\ 0 & -\sin\Phi & \cos\Phi\cos\Theta \end{bmatrix}^{-1} \left[\begin{bmatrix} P^{B/I} \\ Q^{B/I} \\ R^{B/I} \end{bmatrix}_B - \left\{ C^{B/L}(\Phi,\Theta,\Psi) \begin{bmatrix} \dot{\Psi}_{lon}\cos\Theta_{lat} \\ -\dot{\Theta}_{lat} \\ -\dot{\Psi}_{lon}\sin\Theta_{lat} \end{bmatrix}_L \right\}_B \right] \tag{4}$$

In Eq. (4), the second term corresponds to the angular velocity in System E and System L, which is not used in a general aircraft model.

4. Equations of motion for a curved Earth

The System E and System I components of the velocity are denoted by **V** and **U**, respectively. These velocities are expressed by the following equations.

$$\mathbf{V} = \left.\frac{d\mathbf{r}}{dt}\right|_E \tag{5}$$

$$\mathbf{U} = \left.\frac{d\mathbf{r}}{dt}\right|_I = \left.\frac{d\mathbf{r}}{dt}\right|_E + \boldsymbol{\omega}^{E/I} \times \mathbf{r} = \mathbf{V} + \boldsymbol{\omega}^{E/I} \times \mathbf{r} \tag{6}$$

Here, the vector **r** is the position vector from the Earth's center.

The acceleration of the vehicle in the inertial system is defined as shown in the following equation.

$$\left.\frac{d\mathbf{U}}{dt}\right|_I = \left.\frac{d\mathbf{V}}{dt}\right|_I + \boldsymbol{\omega}^{E/I} \times \mathbf{U} = \left(\left.\frac{d\mathbf{V}}{dt}\right|_B + \boldsymbol{\omega}^{B/I} \times \mathbf{V} \right) + \boldsymbol{\omega}^{E/I} \times \left(\mathbf{V} + \boldsymbol{\omega}^{E/I} \times \mathbf{r} \right) \tag{7}$$

Here $\boldsymbol{\omega}^{E/I}$ represents the angular velocity of the Earth's rotation. Thus, $\boldsymbol{\omega}^{B/I}$ is expressed by the relationship shown in Fig. 3.

$$\begin{aligned} \boldsymbol{\omega}^{B/I} &= \boldsymbol{\omega}^{B/E} + \boldsymbol{\omega}^{E/I} \\ &= \boldsymbol{\omega}^{B/L} + \boldsymbol{\omega}^{L/E} + \boldsymbol{\omega}^{E/I} \\ &= \begin{bmatrix} \boldsymbol{i}_B & \boldsymbol{j}_B & \boldsymbol{k}_B \end{bmatrix} \begin{bmatrix} P^{B/L} & Q^{B/L} & R^{B/L} \end{bmatrix}^T \\ &\quad + \begin{bmatrix} \boldsymbol{i}_L & \boldsymbol{j}_L & \boldsymbol{k}_L \end{bmatrix} \begin{bmatrix} 0 & -\dot{\Theta}_{lat} & 0 \end{bmatrix}^T \\ &\quad + \begin{bmatrix} \boldsymbol{i}_L & \boldsymbol{j}_L & \boldsymbol{k}_L \end{bmatrix} \begin{bmatrix} \dot{\Psi}_{lon}\,c\Theta_{lat} & -\dot{\Theta}_{lat} & -\dot{\Psi}_{lon}\,s\Theta_{lat} \end{bmatrix}^T \\ &\quad + \begin{bmatrix} \boldsymbol{i}_E & \boldsymbol{j}_E & \boldsymbol{k}_E \end{bmatrix} \begin{bmatrix} 0 & 0 & \dot{\Psi}_e \end{bmatrix}^T \end{aligned} \tag{8}$$

Thus, the translational motion of the vehicle is described as follows.

$$\left.\frac{d\mathbf{V}}{dt}\right|_B = -\left(\boldsymbol{\omega}^{B/E} + 2\boldsymbol{\omega}^{E/I} \right) \times \mathbf{V} - \boldsymbol{\omega}^{E/I} \times \left(\boldsymbol{\omega}^{E/I} \times \mathbf{r} \right)_B + \left(\mathbf{F}_T + \mathbf{F}_{Aero} + \mathbf{F}_g \right) / m \tag{9}$$

This equation consists of a thrust force $\mathbf{F}_T$, aerodynamics forces including lift, drag, and a side force $\mathbf{F}_{Aero}$, in addition to the gravitational force $\mathbf{F}_g$. On the right hand side of the equation, the first term represents the Coriolis force and the second term represents the centrifugal force. These terms are introduced by considering the rotation and curvature of the Earth, and the components of the Coriolis force and centrifugal force are expressed as shown in the following equations.

$$2\boldsymbol{\omega}^{E/I} \times \mathbf{V} = C^{B/E} \begin{bmatrix} -2\dot{\Psi}_e V_{Ex} & 2\dot{\Psi}_e V_{Ey} & 0 \end{bmatrix}_E^T \tag{10}$$

$$\boldsymbol{\omega}^{E/I} \times \left(\boldsymbol{\omega}^{E/I} \times \mathbf{r} \right) = C^{B/E} \begin{bmatrix} -\dot{\Psi}_e^2 X_E & -\dot{\Psi}_e^2 Y_E & 0 \end{bmatrix}_E^T \tag{11}$$

5. Summary

In this paper, the equations of motion for a space transportation vehicle are derived. Using the proposed model, reentry of the vehicle from an Earth-circular orbit to the ground can be simulated.

6. Reference

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