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Analysis of Electromagnetic Problems with 3-D Surface Models by Curvilinear Patches

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Abstract: Electromagnetic problems with three dimensional (3-D) surface models can be analyzed by using integral solvers. A 3-D object is divided by a number of connected small patches. Computational results obtained by curvilinear patches are compared with those obtained by triangular patches. We show that the number of curvilinear patches is almost a half of that of triangular patches for smooth surfaces under the same computational accuracy.

1. Introduction

Integral solvers can analyze electromagnetic problems with three dimensitonal (3-D) surface models [1][2]. When we use integral solvers, a 3-D object is described by a number of small patches. In this paper, computational results obtained by using curvilinear patches are compared with those obtained by using triangular patches [3]. We show that the curvilinear patch can reduce geometric error when objects have smooth surfaces.

2. Curvilinear Patches

The surface of the sphere is divided by using curvilinear patches or triangular patches as shown in Figure 1. The curvilinear patches are smoothly connected to each other at common boundaries. A curvilinear patch is determined by nine points whose coordinates are represented using two parameters (u_1, u_2). The nine points on the origin of the coordinate system as shown in Figure 2. In this case, a differential tangent vector $d\mathbf{r}$ is given by

$$d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial u_1} du_1 + \frac{\partial \mathbf{r}}{\partial u_2} du_2 . \qquad (1)$$

We consider that the vector $\mathbf{r}(u_1, u_2)$ is second order polynomial in u_1 and u_2. The vector from the origin to the surface of pth patch is written as

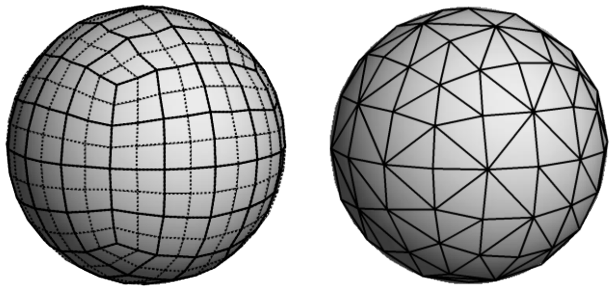
$$\mathbf{r}_p(u_1, u_2) = \sum_{m=1}^{3} \sum_{n=1}^{3} C_{mn}^{(p)} u_1^{m-1} u_2^{n-1} , \qquad (2)$$

where u_1 and u_2 vary from 0 to 1, and $C_{mn}^{(p)}$ are determined from the known nine points coordinates. Thus, the points can be transformed into (u_1, u_2) coordinates as shown in Figure 3.

The differential surface element is given by

$$d\mathbf{S} = \frac{\partial \mathbf{r}}{\partial u_1} \times \frac{\partial \mathbf{r}}{\partial u_2} du_1 du_2 , \qquad (3)$$

where



(a) (b)

Figure 1. Surface discretization.

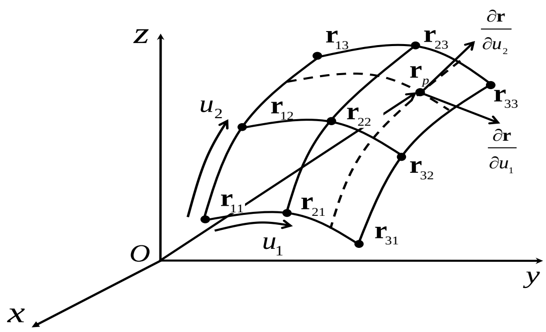


Figure 2. Curvilinear patch.

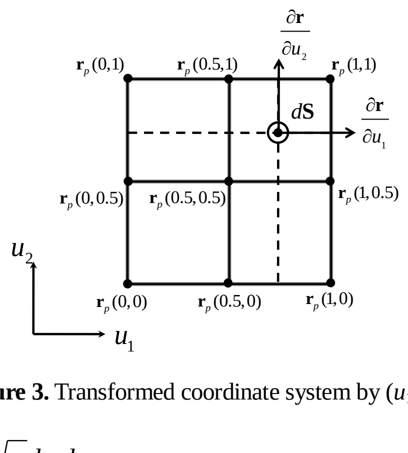


Figure 3. Transformed coordinate system by (u_1, u_2).

$$dS = \sqrt{g}\, du_1 du_2 , \qquad (4)$$

$$g = g_{11} g_{22} - g_{12}^2 , \qquad (5)$$

$$g_{ij} = \frac{\partial \mathbf{r}}{\partial u_1} \cdot \frac{\partial \mathbf{r}}{\partial u_2} . \qquad (6)$$

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3. Computational Results

We inestigate an eigenvalue problem for specific plasmon modes in metallic nanosphere by using the integral solver [2]. Computed eigenvalues are shown in Table 1. The sphere is modeled by 600 curvilinear patches or 1,200 triangular patches. Compared to the analytical Mie series solution, the relative error which is the difference between the analytical value and computational value is less than 1% for all the mode numbers.

Figure 4 shows a charge distribution of plasmon resonant mode for nanosphere. Here, the plasmon mode number is 2. The average lengths of curvilinear patches and triangular patches are same. It is seen that the computational result obtained by curvilinear patches agrees with that obtained by triangular patches.

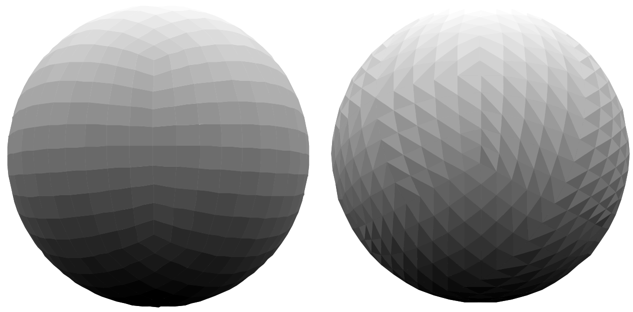
We investigate the computational accuracy for curvilinear patches and triangular patches. Figure 5 shows the convergence test of the eigenvalue whose mode number is 2 for varying the number of patches *N*. Square dots indicate the computational value obtained by using curvilinear patches. Triangle dots indicate that obtained by using triangular patches. We can confirm that the relative error between the computational value and the Mie series solution is less than 1% when the number of patches $N > 500$ for curvilinear patches and the number of patches $N > 1{,}000$ for triangular patches. It is seen that a number of curvilinear patches is almost a half of that of triangular patches.

4. Conclusions

We analyze the electromagnetic problems with 3-D smooth surface models by using the integral solver. The 3-D surface is descritized by using curvilinear patches or triangular patches. We investigate the computational accuracy of curvilinear patches for nanosphere. The number of curvilinear patches is almost a half of that of triangular patches for smooth surfaces under the same computational accuracy.

Table 1. Eigenvalues for a single nanosphere.

Mode Number	Mie Theory	Computed eigenvalues	
		Triangular	Curvilinear
1	3	3.02357688	2.99725748
2	3	3.01910020	3.00750241
3	3	3.02026431	3.00678212
4	5	5.07465290	4.98179931
5	5	5.07465290	5.02563614
6	5	5.05926943	5.02246156
7	5	5.06154918	5.01384330
8	5	5.06460414	5.01179163



(a)　　　(b)

Figure 4. Surface electric charges on a nanosphere.

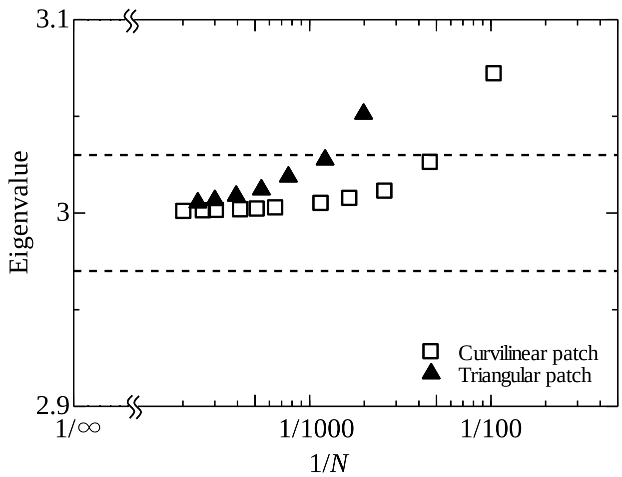


Figure 5. Convergence test of the eigenvalue for varying the number of unknowns *N*.

5. References

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