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Analysis of Large Scale Electromagnetic Scattering Problems by MLFMA
-Prediction and Control of Computational Accuracy-

*Seiya Kishimoto[1], Shinichiro Ohnuki[2]

Abstract: We propose novel error prediction and control methods of the multilevel fast multipole algorithm for electromagnetic scattering problems. The computational error of MLFMA can be predicted without performing any computational tests and can be controlled for optimizing truncation numbers with desired digits of computational accuracy.

1. Introduction

The multilevel fast multipole algorithm (MLFMA) can treat electromagnetic scattering problems with large number of unknowns [1]. However, MLFMA has error sources in the computational process. In this paper, we propose novel error prediction and control methods of MLFMA for electromagnetic scattering problems. Using our method, the computational error for about one million unknowns problems can be precisely controlled under desired digits of accuracy.

2. Formulation

We assume that the scatterers are perfectly conducting objects as shown in Figure 1. The electric field integral equation (EFIE) for conducting objects are given by

$$\hat{t}\cdot\int_S \overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')dS' = \frac{4\pi i}{k\eta}\hat{t}\cdot\mathbf{E}^i(\mathbf{r}), \tag{1}$$

for $\mathbf{r}$ on the surface S, where $\hat{t}$ is a tangential vector on S, $\mathbf{J}(\bullet)$ is unknown surface current distribution, and

$$\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}') = (\bar{\mathbf{I}} - \frac{1}{k^2}\nabla\nabla)g(\mathbf{r},\mathbf{r}'), \tag{2}$$

where $g(\mathbf{r},\mathbf{r}')$ is the scalar Green's function. EFIE is discretized by unknow current distribution expanded in appropriately chosen set of basis function $\mathbf{j}_n(\bullet)$ and testing function $\mathbf{t}_n(\bullet)$. Since this discretization, we can obtain the matrix equation for which to determine unknown current distribution. The computational time and requiring memory for solving matrix equation can be reduced to $O(N \log N)$ by using MLFMA where N is the number of unknowns. To do this, basis functions are divided into the localized groups as shown in Figure 2. The scalar Green's function is expressed by addition theorem [3] and elementary identity such as

$$\frac{e^{ikr_{ji}}}{r_{ji}} \approx \frac{ik}{4\pi}\int d^2\hat{k}\, e^{ik\cdot(\mathbf{r}_{jm}+\mathbf{r}_{m'i})}\sum_{l=0}^{L} i^l(2l+1)h_l^{(1)}(kr_{mm'})P_l(\hat{k}\cdot\hat{r}_{mm'}), \tag{3}$$

where $h_l^{(1)}(\bullet)$ is the spherical Hankel function of the first kind, $P_l(\bullet)$ is the Legendre polynomial, and L is the truncation number. Computational error of MLFMA arises in this truncation process.

The convergence rate of relative error of RCS can be predicted by using the following formula[4-7]:

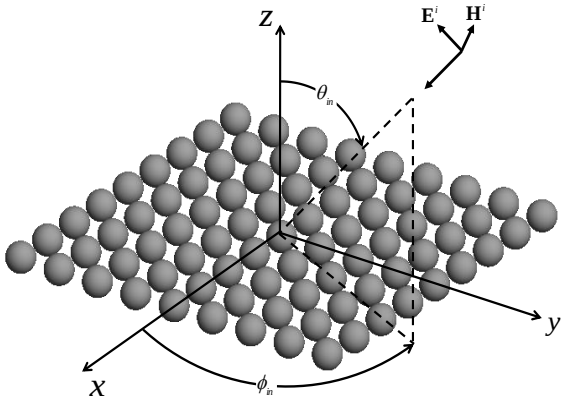

Figure 1. Geometry and coordinate systems.

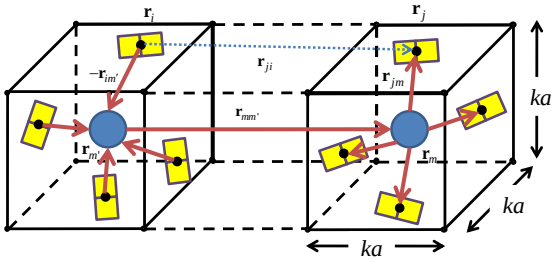

Figure 2. Interaction between element *i* in box *m*' and element *j* in box *m*.

$$C[l] = 10^{-(d_0-d_1-d_2)}, \tag{4}$$

where

$$d_0 := \begin{cases} 0, & L < \sqrt{3}\delta ka \\ \left[\dfrac{L-\sqrt{3}\delta ka}{2.2(\delta ka)}\right]^{3/2}, & \sqrt{3}\delta ka < L < \sqrt{3}\delta ka + 7.1(\delta ka)^{1/3} \\ 6, & \text{otherwise} \end{cases} \tag{5}$$

$$d_1 := \begin{cases} 0, & L < (n+1)ka \\ \left[\dfrac{L-(n+1)ka}{1.8[(n+1)ka]^{1/3}}\right]^{3/2}, & \text{otherwise} \end{cases} \tag{6}$$

1 : Graduate Course of Electrical Engineering, Graduate School of Science and Technology, Nihon University.

2 : Department of Electrical Engineering, College of Science and Technology, Nihon University.

$$d_2 = \log_{10}\left|\frac{\mathbf{t}_j(\mathbf{r})\cdot\mathbf{j}_i(\mathbf{r})}{r_{ji}\max(\bar{\mathbf{A}})}\right| \quad (7)$$

where *ka* is the box size, δ is the interaction parameter, *n* is the number of buffers, and $\max(\bar{\mathbf{A}})$ is the maximum element of the matrix $\bar{\mathbf{A}}$.

3. Computational Results

We perform the convergence test of computational error of MLFMA whose level is 4. The scatterer is the random rough surface. The size of the original flat plate is $6\lambda\times6\lambda$. Figure 3 shows the relative error of the RCS for varying L_4 which is the truncation number of level four. The others level truncation numbers are fixed as optimum values. The lines indicate the predicted error given by Eq. (4). The relative error is defined by the difference of the computational results obtained by using four-level MLFMA and the reference solution. Since MoM cannot be treated large scale problems, the reference solution is considered as the RCS obtained by three-level MLFMA with optimum selection of truncation numbers. The convergence process of the relative error follows the solid line. Therefore the computational error can be predicted by using Eq. (4).

We verify our method for a large scale problem. The scatterer consists of 35×35 conducting spheres modeled by 470,400 curvilinear patches with 940,800 unknowns on *x-y* plane as shown in Figure 1. The bistatic RCS computed by using seven-level MLFMA is plotted in Figure 4. The truncation numbers of all the levels are selected to achieve the desired digits of accuracy *d* = 1E-3 using Eq. (4). We can confirm that the RCS is enhanced around $\theta=30^\circ$. Compared with the reference solution obtained by six-level MLFMA for the optimum selection of truncation numbers, the relative error can be fully controlled for all the observation angles in Figure 5.

4. Conclusions

We have studied error analysis of MLFMA for electromagnetic scattering problems and propose the error prediction and control methods. The computational error can be estimated by using our prediction method. Using this technique, the computational error of large scale electromagnetic problems can be controlled under desired digits of accuracy.

5. Acknowledgements

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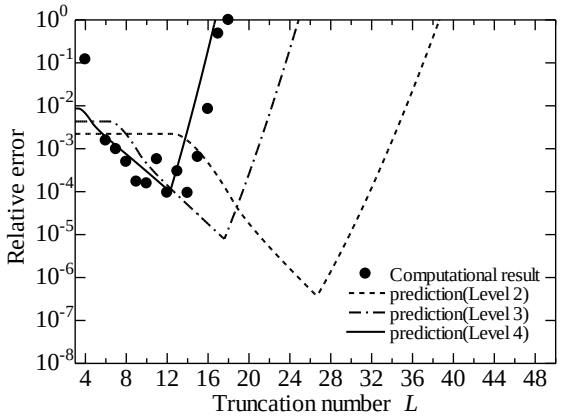

Figure 3. Convergence test of the relative error for varying the number of unknowns *N*.

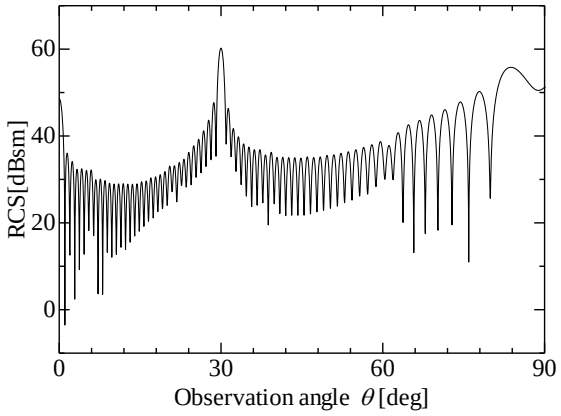

Figure 4. Bistatic RCS of many conducting spheres.

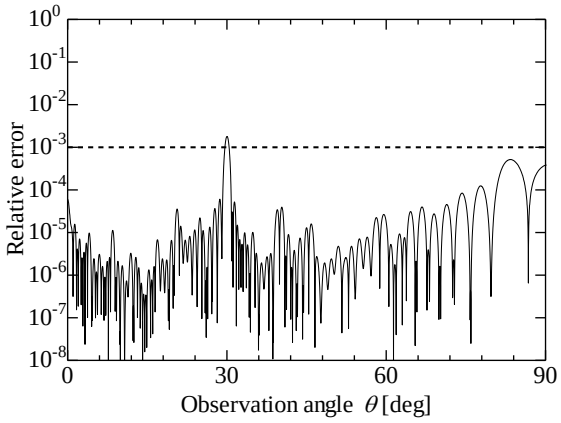

Figure 5. Relative error of the bistatic RCS for many spheres.

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