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Computational Accuracy of the Three Dimensional Point Matching Method for a PEC Sphere

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Abstract : The point matching method can analyze electromagnetic scattering problems with a high degree of accuracy. In this paper, we propose the way to predict the computational accuracy of the three dimensional point matching method for a PEC sphere. Computational accuracy due to the placement of sampling points is discussed.

1. Introduction

Electromagnetic scattering problems can be analyzed with a high degree of accuracy by using the point matching method (PMM)[1]. Recently, we have proposed prediction of the computational accuracy for the polygonal cylinder[2][3]. In this paper, we propose the way to predict the computational accuracy of the three dimensional point matching method for a PEC sphere. Computational accuracy due to the placement of sampling points is clarified.

2. Formulation

The geometry of the scatterer is shown in Figure 1. The incident wave is a plane wave propagating toward $+z$ direction. The components of the electric fields in θ and ϕ directions can be written as

$$E_\theta^i = E_0 \cos\theta \cos\phi \, e^{-jkr\cos\theta} \quad , \qquad (1)$$

$$E_\phi^i = -E_0 \sin\phi \, e^{-jkr\cos\theta} \quad . \qquad (2)$$

The time dependence is $e^{j\omega t}$ and suppressed throughout the paper.

In our PMM, we need to divide the whole physical space into regions to satisfy the wave equation. The electromagnetic field of the outside of the sphere is expressed as follows;

$$E_\theta^S = \frac{E_0}{kr}\cos\phi \sum_{n=1}^{N}\left[b_n \hat{H}_n^{(2)\prime}(kr)\sin\theta \, P'^1_n(\cos\theta) - c_n \hat{H}_n^{(2)}(kr)\frac{P_n^1(\cos\theta)}{\sin\theta}\right] , \qquad (3)$$

$$E_\phi^S = \frac{E_0}{kr}\sin\phi \sum_{n=1}^{N}\left[b_n \hat{H}_n^{(2)\prime}(kr)\frac{P_n^1(\cos\theta)}{\sin\theta} - c_n \hat{H}_n^{(2)}(kr)\sin\theta \, P'^1_n(\cos\theta)\right] , \qquad (4)$$

where $\hat{H}_n^{(2)}(kr) = \sqrt{\pi kr/2}\cdot H_{n+1/2}^{(2)}(kr)$, $H_n^{(2)}(\bullet)$ is the n-th order of the second kind of the Hankel function, N is the truncation mode number, $P_n^1(\bullet)$ is n-th order of the Legendre function, b_n and c_n are the unknown expansion coefficients, and k is the wave number.

The unknown coefficients b_n and c_n are determined to satisfy the continuity conditions at the sampling points which are placed in θ and ϕ directions.

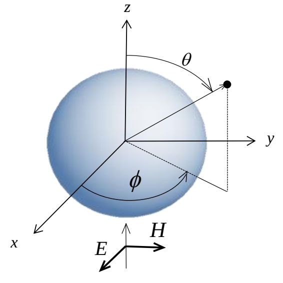

Figure 1. Geometry of the scatterer.

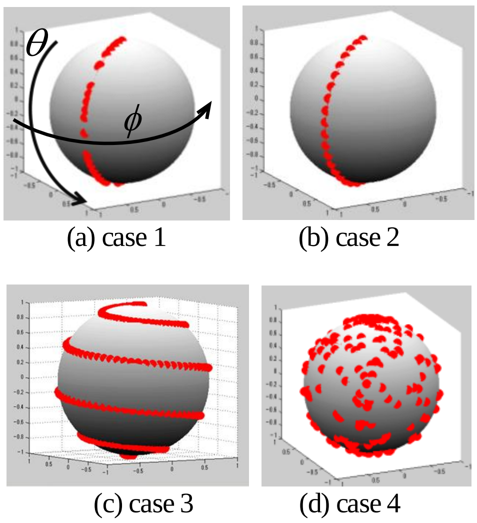

Figure 2. Placement of sampling points.

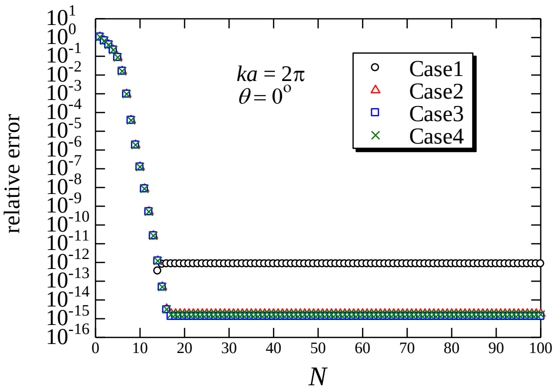

Figure 3. Computational accuracy for the truncation mode number N.

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3. Computational Results

To study the computational accuracy of PMM, the following four cases shown in Figure 2 about the selection of intervals of the sampling points $\Delta\theta_n$ and $\Delta\phi_n$ are investigated;

(a) Case 1: $\begin{cases} \Delta\theta_n : 180^\circ n \times rand(n) \\ \Delta\phi_n : 0 \end{cases}$,

(b) Case 2: $\begin{cases} \Delta\theta_n : 180^\circ n/(N+1) \\ \Delta\phi_n : 0 \end{cases}$,

(c) Case 3: $\begin{cases} \Delta\theta_n : 180^\circ n/(N+1) \\ \Delta\phi_n : 360^\circ n/(N+1) \end{cases}$,

(d) Case 4: $\begin{cases} \Delta\theta_n : 180^\circ n/(N+1) \\ \Delta\phi_n : 360^\circ n \times rand(n) \end{cases}$,

where $n = 1 \sim N$.

Figure 3 plots computational accuracy for varying the truncation mode number *N*. Compared with case 1, we can obtain higher accuracy for case 2. Cases 2, 3 and 4 are almost same results. To achieve higher accuracy, sampling points should be placed at the same interval in θ direction.

Figures 4 and 5 show the convergence test of E_θ^s and E_ϕ^s for varying the mode number *n*. The dashed line indicates the error prediction given by[2] [3]

$$C_J = 10^{-d_0} , \tag{5}$$

$$d_0 = \left\{ \frac{N - ka}{1.8\,(ka)^{1/3}} \right\}^{3/2} , \tag{6}$$

where $N > ka$.

All the results are in excellent agreement.

Figure 6 plots the convergence test of the relative error for varying the truncation mode number *N*. The characteristic of convergence for PMM and the error prediction is in good agreement. Therefore, computational error of PMM can be predicted by using Eq.(5).

4. Conclusions

We have proposed the way to predict the computational accuracy of the three dimensional point matching method for a PEC sphere. Computational accuracy for placement of sampling points has been discussed. Considering computational results, the relative error for varying the truncation mode number *N* can be predicted by using the prediction equation.

5. Ackowledgemetns

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6. References

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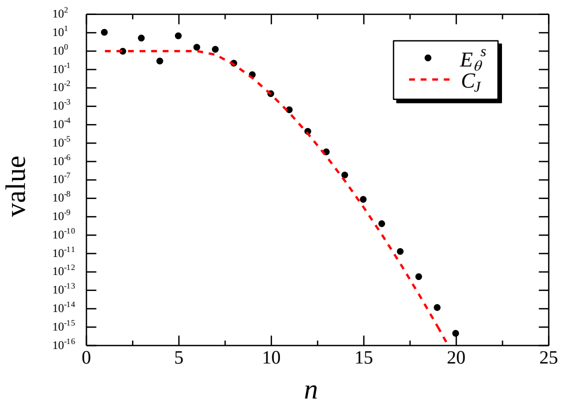

Figure 4. Convergence test of E_θ^s for varying the mode number *n*

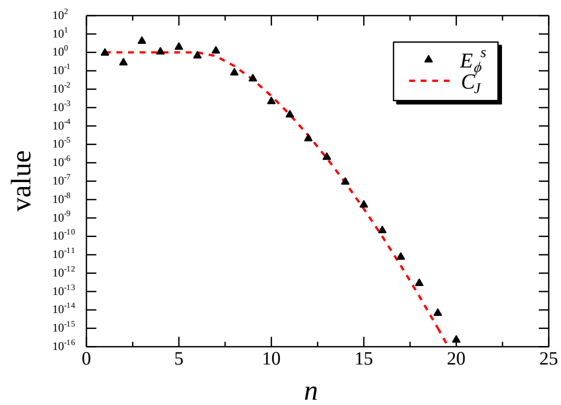

Figure 5. Convergence test of E_ϕ^s for varying the mode number *n*

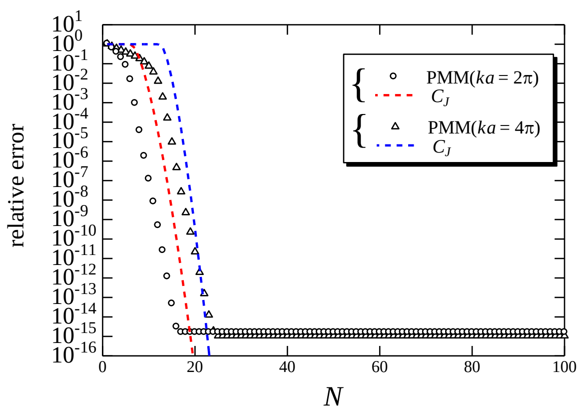

Figure 6. Convergence test of the relative error for varying the truncation mode number *N*.

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