K3-16

# Calculation of Particle Motion in Ultrasonic Standing Wave

*Hiroaki Mori[1], Kenji Kofu[2], Mitsuaki Ochi[2]

Abstract: Purpose of this research is to analyze minute sized particles motion in air while ultrasonic standing wave is generated by flexural vibration plate. Firstly, it is necessary to obtain that the acoustic pressure distribution of standing wave. Then, standing wave of one dimension is calculated simply from theoretical equation. And acoustic radiation force which acts on particles was derived from the obtained pressure. Finally, movement of particles was calculated by computational fluid dynamics. As a result, particles gathered at the node of standing wave by acoustic radiation force. This also shows particles can be concentrated early as acoustic radiation force becomes higher.

## 1. INTRODUCTION

Dr. Kofu [1] had shown that the ultrasonic can reduce the frictional resistance of static particles on an ultrasonic vibration flat plate and dynamic particles for horizontal plug conveying. In this research process, particles are sprinkled uniformly on the surface of vibration plate. It was observed that scattered particles are gathered at node of vibration plate. Kozuka [2] proved that particles behavior were controllable by using ultrasonic in water. It is thought that nano-sized particles in air can be gathered to any position by applying this phenomenon into the air. Then authors propose a new method of dust collection, that is using ultrasonic standing wave. In this time, it’s hard to investigate the effect of this method experimentally and enormous cost is required. If we can calculate dusting minute sized particle motion by the ultrasonic, we can reduce the cost and time of the experiment. In this research, minute sized particle motion is calculated by computational fluid dynamics (CFD) in ultrasonic standing wave.

## 2. THEORY

### 2.1 Government equation

PHOENICS (software of thermal fluid analysis) for CFD is used. When particle motion is calculated, government equation of continuous phase are applied.

$$\frac{\partial}{\partial t}(\rho_c\phi_c)+\nabla\cdot(\vec{U}_c\rho_c\phi_c)-\nabla\cdot(\Gamma_{\phi c}\nabla\phi_c)=S_\phi+S_{\phi G} \qquad (1)$$

$\Gamma_{\varphi c}$ is exchange coefficient, $S_\varphi$ is generation term, $S_{\varphi G}$ is a term which represents the effect of the particles applied on the continuous phase. Particle effects for the continuous phase are considered in this section. Equation (1) is discretized by a full-implicit method or hybrid method. Then, two-dimensional discretization equation summarizes the steady-state term, advection term, diffusion term, and generation term is obtained by the number of cells. Each variable are calculated by the iterative calculation such as the Jacobi method.

### 2.2 Motion equation of particle

The following shows the particle motion equation.

$$m_p\frac{d\vec{U}_p}{dt}=\frac{1}{2}\rho_c A_p C_D\left|\vec{U}_c-\vec{U}_p\right|\cdot(\vec{U}_c-\vec{U}_p)+m_p b\vec{g}+F_{ac} \qquad (2)$$

$m_p$ is the mass of the particle, $\vec{U}_c$ and $\vec{U}_p$ are medium and particle velocity, $C_D$ is drag coefficient. Also, Drag coefficient is used for the following Stokes equation.

$$C_D=\frac{24}{\mathrm{Re}} \qquad (3)$$

$b$ is buoyancy factor and treated as 1.0 because it isn’t considered the buoyancy. In addition, $F_{ac}$ is acoustic radiation force, described below.

### 2.3 Acoustic radiation force

Acoustic radiation force that acts on spherical particles in ultrasonic standing wave is represented by the following formula [3]. However, it can be used only if the particle size is sufficiently small compared to the wavelength.

$$\overline{F_{ac}}=V_p(D\nabla\overline{e_k}-(1-\gamma)\nabla\overline{e_p}),\ \ D=3(\rho_p-\rho_0)/2(\rho_p+\rho_0)$$

$$\overline{e_p}=\frac{\overline{p_a^2}}{2\rho_0 {c_0}^2} \qquad \overline{e_k}=\frac{\rho_0\overline{v_a^2}}{2} \qquad (4)$$

$V_p(=4\pi a^3/3)$ is volume of the particle, $\gamma$ is compression ratio of the particle and the medium. $\rho_p$ and $\rho_0$ are particles and medium density. $\bar{e}_k$ and $\bar{e}_p$ are the gradient of the time average value of the kinetic energy and potential energy.

### 2.4 Sound pressure in one-dimensional standing wave

For simplicity, the sound pressure in the one-dimensional standing wave is used. It can be represented by the following equation (5) theoretically.

$$P_{ac}=P_{ac\max}\cos kx\cdot\sin(2\pi ft) \qquad (5)$$

$P_{acmax}$ is maximum sound pressure, $k$ is wave number ($k=2\pi/\lambda$), $f$ is frequency, $x$ is the length from the radiation, and $t$ is time.

## 3. CALCULATION CONDITION

### 3.1 Analysis domain and condition

Fig.1 shows the analysis area. The end face is treated as free outflow and the vibration plate was placed as shown in the figure. Two plates are placed in the analysis domain but they don’t vibrate. Because, the acoustic radiation force in one-dimensional standing wave is input in advance. Table 1 shows the size of the area and the number of grids. Table 2 shows the conditions of each parameter.

Table 1 Computational grid

| | *x* | *y* | *z* |
|---|---|---|---|
| cell number | 2000 | 20 | 1 |
| size [mm] | 1000 | 10 | 1 |

Table 2 Analysis condition

| | |
|---|---|
| Domain fluid | compressible air |
| turbulent model | laminar flow |
| Fluid temperature $T$ [℃] | 20 |
| Fluid density $\rho_0$ [mg/mm$^3$] | 0.0012 |
| Particle dendity $\rho_p$ [mg/mm$^3$] | 1 |
| Vibration frequency $f$　[kHz] | 34 , 68 , 102 |
| Max fluid velocity $U_{max}$ [m/s] | 0.1 |

---

1:Department of Mechanical Engineering, Graduate School of Science & Technology, Nihon University
2：Department of Mechanical Engineering, College of Science & Technology, Nihon University

### 3.2 Definition of particle concentration

It is necessary to establish the definition for the particles concentration. When the distance between the node of the standing wave and the particles is 1μm, it is assumed as the particles are concentrated. In addition, particles are injected at inlet, and necessary concentration length in flow direction $L_c$ is calculated.

## 4. CALCULATION RESULTS AND CONSIDERATION

### 4.1 Sound pressure distribution

Fig.2 shows the sound pressure distribution was calculated using the equation (5). Frequency is 68kHz and the maximum sound pressure is 2kPa. Sound pressure at node is almost 0kPa and anti-node is maximum or minimum.

### 4.2 Calculation of particle motion

*Lc* was calculated in various frequency parameters, the maximum sound pressure and particle size. Fig.2 shows the 12 particle motion. It is injected at intervals of 0.2mm in y direction. It is easy to find out that particles are concentrated at the node of acoustic pressure.

### 4.3 Relation of maximum pressure and length of particles concentration

Fig.3 shows the relation of $P_{acmax}$ and $L_c$. $L_c$ was calculated by changing particle diameter for $d_p$=0.5 and 1.0μm at frequency $f$=68kHz. It reveals that becomes shorter as particle size is larger. This is because the acoustic radiation force increases as the particle size increases. Therefore, it can be said that becomes difficult as particle size is smaller.

In addition, the distance is shortened by increasing the concentration sound pressure. However, this tendency is attenuated. It might be considered the maximum sound pressure in response to a particle size and frequency.

### 4.4 Relation of particles diameter and length of particles concentration.

Fig.3 shows the relation of $d_p$ and $L_c$. $L_c$ was calculated by changing maximum pressure $P_{acmax}$=2, 6 and 10kPa at frequency $f$=68kHz. Length of particles concentration is longer with a smaller particle size. Therefore, the reduction of acoustic radiation force make difficult to collecting minute particles. The difference between $P_{acmax}$=6 and 10kPa is small comparing with $P_{acmax}$=2 and 6kPa. As described above, it might be considered the maximum sound pressure in response to a particle size and frequency.

## 5. CONCLUSION

1) It was able to verify that the particles are concentrated to a node of the standing wave analysis.
2) Under higher maximum sound pressure, particle size and frequency, the larger acoustic radiation force can be obtained.
3) It is difficult to collect minute particles by the acoustic radiation force. Suitable acoustic pressure distribution should be considered in order to collect minute particles.

## 6. REFERENCES

1) K. Kofu, M. Ochi, H. Miura and Y. Hirai, “Influence of Particle Properties on Friction Reduction of Particle Mass by Ultrasonic”, *J.Soc.Powder Technol.,Japan*, 47, 4, 222-229, 2010.
2) T. Kozuka, H. Yasui, T. Tsuziuti, A. Towata and Y. Iida, “Measurement and Calculation of the Radiation Force on a Small Object in a Standing Wave Field”, *The Transactions of the Institute of Electronics, Information and Communication Engineers. A*, J91-A(12), 1156-1160, 2008.
3) Edited by Chouonpa binran hensyuiinkai, ”Chouonpa binran”, 196-198, 1999 Maruzen Company, Limited.

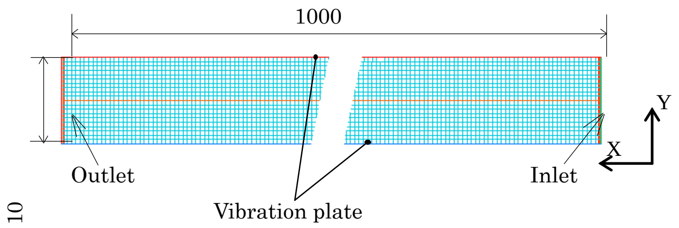

Fig.1 Calculation domain (units in mm)

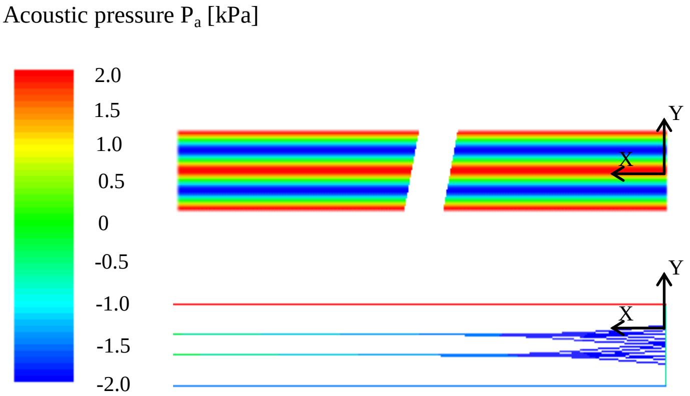

Fig.2 Pressure distribution of one dimension and particles movement

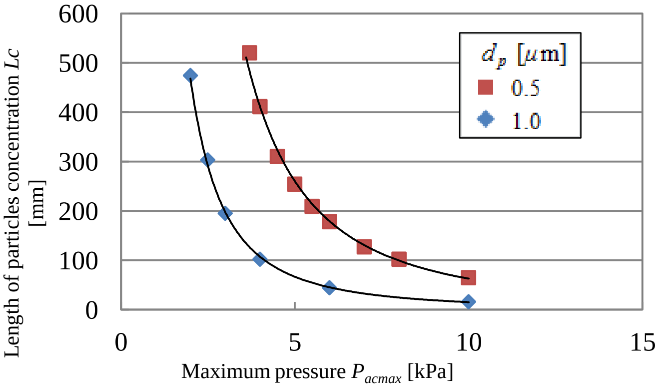

Fig.3 Relation of maximum pressure and length of particles concentration

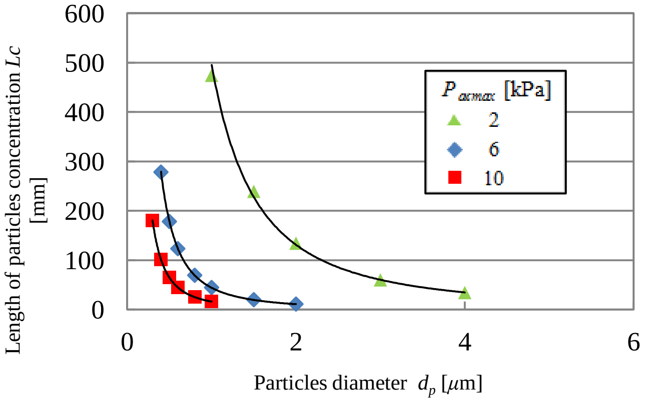

Fig.4 Relation of particles diameter and length of particles concentration