# Vector Fields for UAV Guidance Using Potential Function Method 

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#### Abstract

This paper describes a design of guidance law for a fixed－wing unmanned aerial vehicle（UAV）．A vector field， which is generated by using the potential function method，yields globally stable tracking of geometrical patterns．A guidance of a UAV to an observation circle around a target is determined by building a vector field that has a stable limit cycle centered on the target position．The guidance law is obtained by considering a vector field as velocity field for UAV dynamics．The guidance law and the optimal control theory are applied to design of a flight control system for a small UAV． Numerical simulation is performed to verify the validity of the proposed flight control system．Experiments are also demonstrated by using the proposed guidance law．


## 1．Introduction

The vector field based approache have been extensively used to uide and control vehicles ${ }^{[1]}$ in the execution of tasks．We have designed a guidance law ${ }^{[2]}$ ，which is based on the potential function method，for a fixed－wing UAV．The UAV must maintain significant forward flight speed in order to remain aloft and have a possible turning radius for disastar monitoring or atmospheric observation．The guidance law， however，generated sometimes unsatable command when the UAV across equilibrium state defined by a potential function． Furthermore，a flight control system was unable to maintain altitude of the UAV．

In this paper，we describe the design method of guidance law using the potential function method and flight control system considering altitude control．A velocity field that is generated by a potential function is redesighed to obtain stable commands by applying a rotating potential function．An optimal regulator is designed for rateral motion and longitudinal motion that takes a quantity relating to the altitude of the UAV into account．In addition，we report the results calculated by numerical and experimental results．

## 2．Aircraft Dynamics and Control

Figure 1 shows the definitions of the state variables and control inputs of the UAV．


Figure 1 Definition of State Variables and Control Inputs
Assuming that the motion of the UAV is within the linear range，the equation of motion of the UAV is expressed as follows：

$$
\begin{align*}
\frac{d}{d t}\left[\begin{array}{c}
u \\
w \\
z \\
q \\
\theta
\end{array}\right] & =\left[\begin{array}{ccccc}
X_{u} & X_{w} & 0 & -W_{0} & -g \cos \theta_{0} \\
Z_{u} & Z_{w} & 0 & U_{0}+Z_{q} & -g \sin \theta_{0} \\
0 & 1 & 0 & 0 & 0 \\
M_{u} & M_{w} & 0 & M_{q} & 0 \\
0 & 0 & 0 & 1 & 0
\end{array}\right]\left[\begin{array}{c}
u \\
w \\
z \\
q \\
\theta
\end{array}\right]+\left[\begin{array}{cc}
0 & X_{\delta} \\
Z_{\delta e} & 0 \\
0 & 0 \\
M_{\delta e} & M_{\delta} \\
0 & 0
\end{array}\right]\left[\begin{array}{c}
\delta e \\
\delta t
\end{array}\right](1  \tag{1}\\
\frac{d}{d t}\left[\begin{array}{c}
v \\
p \\
r \\
\varphi \\
\psi
\end{array}\right] & =\left[\begin{array}{ccccc}
Y_{v} & Y_{p} & Y_{r}-U_{0} & g \cos \theta_{0} & 0 \\
L_{v} & L_{p} & L_{r} & 0 & 0 \\
N_{v} & N_{p} & N_{r} & 0 & 0 \\
0 & 1 & \tan \theta_{0} & 0 & 0 \\
0 & 0 & \sec \theta_{0} & 0 & 0
\end{array}\right]\left[\begin{array}{l}
v \\
p \\
r \\
\phi \\
\psi
\end{array}\right]+\left[\begin{array}{cc}
Y_{\delta a} & Y_{\delta r} \\
L_{\delta a} & L_{r} \\
N_{\delta a} & N_{\delta r} \\
0 & 0 \\
0 & 0
\end{array}\right]\left[\begin{array}{l}
\delta a \\
\delta r
\end{array}\right](2) \tag{2}
\end{align*}
$$

$z$ of the body axis coordinate is added to Eq．（1）as a new state variable to accomplish altitude control．Here，$\varphi, \theta$ ，and $\psi$ are Euler angles，$p, q$ ，and $r$ are angular rates，$\delta_{a,} \delta_{e}, \delta_{r}$ ，and $\delta_{t}$ are deflections of the aileron，elevator，rudder，and throttle， respectively shown in Fig．1．It is confirmed that the equations above are controllable and observable．

The feedback gains are determined by using the optimal regulator．It should be noted that state quantities，velocities $u$ ，$v$ and $w$ were estimated by using the observer，and the observer gain was determined using the pole placement method．

## 3．Guidance Law

The potential function method is used to guide the UAV to a desired position．The derivative of the potential field is applied to the UAV＇s velocioty field．The potential function consists of a steering potential $U^{S}$ and a rotating potential $U^{\mathrm{C}}$ ．Velocity vector is defined as following equations．

$$
\begin{align*}
& u_{x}=-\frac{\partial U^{S}(x, y)}{\partial x}+\frac{\partial U^{C}}{\partial y}  \tag{3}\\
& u_{y}=-\frac{\partial U^{S}(x, y)}{\partial y}-\frac{\partial U^{C}}{\partial x} \tag{4}
\end{align*}
$$

where $x$ represents the position vector．The steering potential function $U^{S}$ are defined by the following equations．

$$
\begin{equation*}
U^{S}\left(x_{i}, y_{i}\right)=C_{h} \sqrt{\left(\sqrt{x_{i}^{2}+y_{i}^{2}}-r\right)^{2}+1} \tag{5}
\end{equation*}
$$

[^0]The function $U^{S}$ guides the UAV on a circle of radius $r$ ，and $C_{h}$ is a coefficient which determines the rate of change of the induced velocity．In addition，the rotating potential function when performing circular flight is defined as below．

$$
\begin{equation*}
U^{c}\left(x_{i}, y_{i}\right)=\frac{C_{c}}{1+\exp \left(L_{c}\left(\sqrt{x_{i}^{2}+y_{i}^{2}}-r\right)\right)} \tag{6}
\end{equation*}
$$

where $C_{c}$ is the magnitude and direction of the rotation vector， $L_{c}$ describes the area influenced．Using this velocity field，the command for heading angle $\psi$ and command velocity of the UAV are defined below．

$$
\begin{equation*}
\psi_{d}=\tan ^{-1}\left(u_{y} / u_{x}\right) \quad u_{d}=\sqrt{u_{x}^{2}+u_{y}^{2}} \tag{7}
\end{equation*}
$$

4．Numerical Simulation
A numerical simulation of a circular flight pattern of a single UAV was performed to demonstrate the effectiveness of the method proposed．


Figure 2 Numerical Simulation Results

The numerical simulation results are shown in Fig．2．Target altitude is set at $70[\mathrm{~m}]$ ，and the guidance radius is set to $50[\mathrm{~m}]$ ． The simulation result shows that the UAV draws a circle with a radius of $50[\mathrm{~m}]$ ，while rising up to and maintaing the target altitude of $70[\mathrm{~m}]$ ．

## 5．Experiment of Altitude Control

We have developed a UAV shown in Fig． 3 to verify this method with the use of an actual device．The UAV is equipped with an IMU，Microcomputer GPS and Radio Module．The IMU detect the attitude angle and angular rate．The GPS measures the current position．The Microcomputer calculates the control low．The Radio module is used to send data to a Ground Station．Altitude data are obtained from the GPS．GPS data are the central principal axis coordinate．


A guidance and control experiment was performed with single UAV in circular flight．The reason of circular flight is narrow flight area of the experiment field．So this experiment was performed in circular flight．The experimental conditions were same as the numerical simulation．In addition，takeoff of the UAV was performed manually，and was then switched to autonomous flight control．The experimental results are shown in Figure 4.


Figure 4 （a）shows the $\varphi$ and $\theta$ angles of the UAV．The $\varphi$ and $\theta$ angles are trying to maintain a constant value．In particular，$\theta$ value is keeping a constant value，because of UAV is maintained target altitude．Figure 4（b）shows the $\psi$ and $\psi_{d}$ angles of the UAV．The angle $\psi$ follows its command value． Figure 4（c）shows the altitude of the UAV．The altitude is maintained within $5[\mathrm{~m}]$ with respect to the target altitude． Furthermore，Figure 4（d）shows the trajectory of the UAV．It is obvious that the UAV is turning to the circle．From these results，the effectiveness of the proposed method was confirmed．

## 6．Conclusion

Circular flight due to potential function including altitude control was verified from the numerical simulation results and demonstrative experiment．Future works include achieving demonstrative experiments of formation flight by incorporating the proposed method with improved．

## References

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