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Analysis of Electromagnetic Problems for a Dielectric Sphere by PMCHWT

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Abstract: The PMCHWT (Poggio-Miller-Chang-Harrigton-Wu-Tsai) formulation is one of integral equation formulations for analysis of electromagnetic problems with a homogenous dielectric object. In this paper, the PMCHWT formulation is expanded to the complex frequency domain. Electromagnetic scattering problems for a dielectric sphere are investigated both frequency and time domains.

1. Introduction

Analysis of electromagnetic problems for dielectric objects is important for a wide variety of applications such as design of nanoscale devices [1].

In this paper, we investigate the electromagnetic scattering problems for a dielectric sphere by the method of moments (MoM) based on the PMCHWT (Poggio-Miller -Chang-Harrigton-Wu-Tsai) formulation in the complex frequency domain [2]. The MoM is applied the fast inverse Laplace transform (FILT) [3, 4]. The computational results are compared with the exact solutions both the frequency and time domain.

2. Formulation

We assume that the scatterer is a homogeneous dielectric sphere as shown in Figure 1. The sphere has the permittivity ε_2 and the permeability μ_2. The electric field integral equation (EFIE) and the magnetic field integral equation (MFIE) are given by using the equivalent principle and the extinction theorem[2]. Considering the field equivalence principle, the equivalence surface electric current $\mathbf{J}_\Omega$ and magnetic current $\mathbf{M}_\Omega$ are given by

$$\mathbf{J}_\Omega = \hat{n}\times\mathbf{H},\quad \mathbf{M}_\Omega = \mathbf{E}\times\hat{n} \tag{1}$$

where $\mathbf{E}$ and $\mathbf{H}$ represent total fields and $\hat{n}$ is the outward normal unit vector on surface Ω. To consider the boundary condition, the tangential compornents of the electric and magnetic fields are continuous at the boundary, the PMCHWT formulation in the complex frequency domain can be represented by

$$-\hat{t}\cdot\mathbf{E}^i(\mathbf{r}) = \hat{t}\cdot\sum_{m=1}^{2}\left[L_m(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_\Omega(\mathbf{r}') - K_m(\mathbf{r},\mathbf{r}')\cdot\mathbf{M}_\Omega(\mathbf{r}')\right], \tag{2}$$

$$-\hat{t}\cdot\mathbf{H}^i(\mathbf{r}) = \hat{t}\cdot\sum_{m=1}^{2}\left[K_m(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_\Omega(\mathbf{r}') + \frac{1}{\eta_m^2}L_m(\mathbf{r},\mathbf{r}')\cdot\mathbf{M}_\Omega(\mathbf{r}')\right], \tag{3}$$

where the L_m and K_m are integral operators defined by

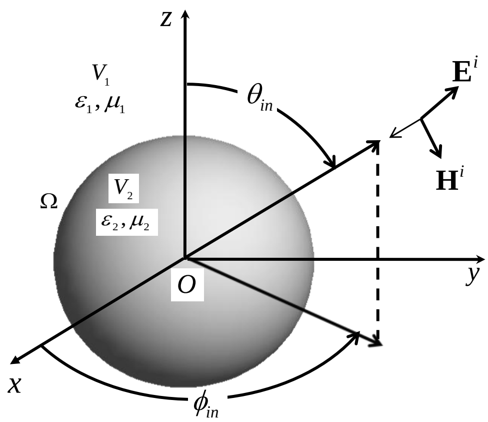

Figure 1. Geometry and coordinate systems.

$$L_m(\mathbf{r},\mathbf{r}')\cdot\mathbf{X}(\mathbf{r}') = -s\mu_m\int_\Omega\left[\mathbf{X}(\mathbf{r}') - \frac{\nabla\nabla}{S_m^2}\mathbf{X}(\mathbf{r}')\right]g_m(\mathbf{r},\mathbf{r}')d\Omega', \tag{4}$$

$$K_m(\mathbf{r},\mathbf{r}')\cdot\mathbf{X}(\mathbf{r}') = \int_\Omega\mathbf{X}(\mathbf{r}')\times\nabla g_m(\mathbf{r},\mathbf{r}')d\Omega', \tag{5}$$

$$g_m(\mathbf{r},\mathbf{r}') = \frac{\exp(-S_m|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|}, \tag{6}$$

$\hat{t}$ is the tangential unit vector on the surface Ω, s is the complex frequency, $S_m = s\sqrt{\varepsilon_m\mu_m}$, and $\eta_m = \sqrt{\mu_m/\varepsilon_m}$.

In the MoM, to solve the Eqs. (2) and (3), the unknown electric and magnetic current distributions are discretized by basis functions $\mathbf{f}_n$, such as

$$\mathbf{J}(\mathbf{r}) = \sum_{n=1}^{N}J_n\mathbf{f}_n(\mathbf{r}),\quad \mathbf{M}(\mathbf{r}) = \sum_{n=1}^{N}M_n\mathbf{f}_n(\mathbf{r}), \tag{7}$$

where J_n and M_n are the unknown expansion coefficitents and N is number of unknowns. For the basis function, we chose the RWG (Rao-Wilton-Glisson) function [2]. Eqs. (2) and (3) are discretized by using Eq. (7) and testing function $\mathbf{t}_n$. The testing function is same as the basis function. Hence, we can obtaine the $2N\times 2N$ matrix equation. The unknown expansion coefficitents can be found by solving matrix equation.

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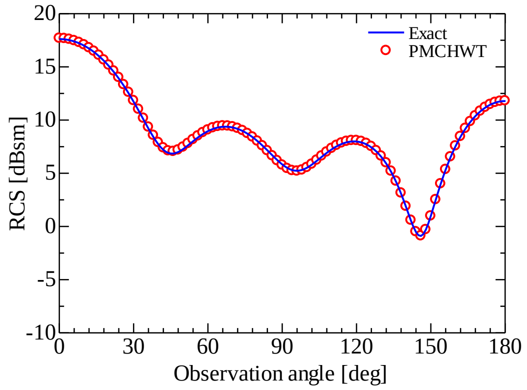

Figure 2. Bistatic RCS for a dielectric sphere.

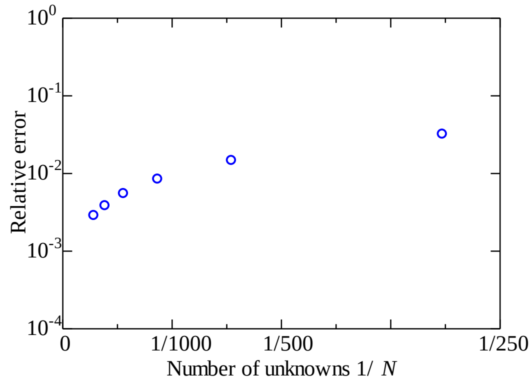

Figure 4. Convergence test of the relative error for varying the number of unknowns *N*.

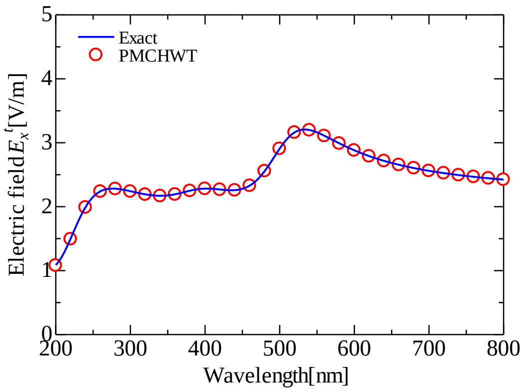

Figure 3. Wavelength response for a gold sphere.

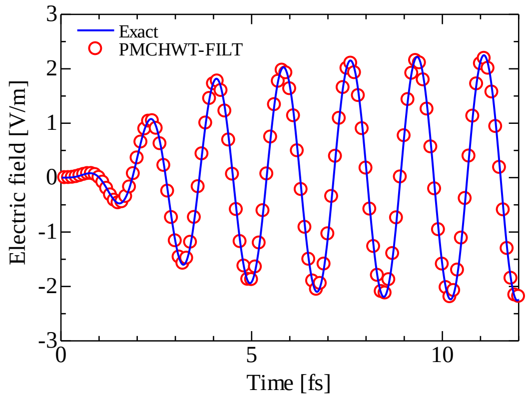

Figure 5. Time domain response for a gold sphere.

3. Computational Results

Figure 2 shows the radar cross section (RCS) for a dielectric sphere with radius r = 0.5λ and the relative permittivity $\varepsilon_r = 4.0$. The incident wave impinges from $\theta_{in} = 180^\circ$ and $\phi_{in} = 0^\circ$. The computational results and the exact solution given by the Mie series are in an excellent agreement.

We investigate the electric field near a gold nanosphere with the radius 25.0 nm. The observation point is at x = 37.5 nm on the x-axis. The relative permittivity for a gold nanosphere is expressed by the Lorents-Drude model [4]. Figure 3 shows the wavelength response of the electric field. The computational result corresponds to the exact solution. The convergence test of the relative error for varying the number of unknowns N is shown in Figure 4. The relative error is defined by the difference between the computational result and the exact value when wavelength is 525nm. The relative error is less than the 1.0% for $N > 1000$.

Figure 5 shows the time domain response of the electric field. The incident wavelength is 525 nm. For analysis of the time domain response, the functions in the complex frequency domain are transformed into the time domain by applying the FILT [3, 4]. We confirmed that the relative error is less than 1.0% at all observation times.

4. Conclusions

Electromagnetic problems for a dielecctric sphere are investigated by the MoM based on the PMCHWT formulation in the complex frequency domain. FILT has been applied to the MoM. The computational results agree vely well with the exact solution.

5. Acknowledgements

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6. References

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