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Computational Accuracy of the 3D Point Matching Method for Analyzing Electromagnetic Scattering from a Spherical Shell

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Abstract : Electromagnetic scattering problems can be analyzed with a high degree of accuracy by using the point matching method. Recently, we have studied computational accuracy of scattering problems for perfect electric conducting and dielectric spheres as examples of 3D canonical geometries. In this paper, we investigate computational accuracy of the 3D point matching method for analyzing electromagnetic scattering from a spherical shell.

1. Introduction

The authors have proposed the point matching method (PMM) which can analyze electromagnetic scattering problems with high degree of accuracy [1]. Recently, we have studied computational accuracy of scattering problems for perfect electric conducting (PEC) and dielectric spheres as examples of 3D canonical geometries for 3D-PMM [2].

In this paper, we investigate computational accuracy of the 3D PMM for analyzing electromagnetic scattering from a spherical shell.

2. Formulation

Figure 1 shows the geometry of a scatterer. The incident wave is a plane wave propagating toward $+z$ direction. The scatterer is composed of a dielectric sphere whose radius is a and the PEC shell which is zero thickness and covered in the range of $\theta_1 \le \theta \le \theta_2$.

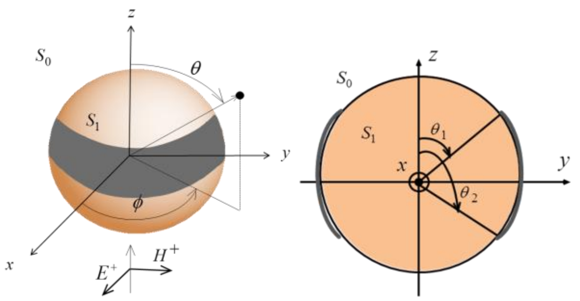
The θ and ϕ components of the incident electric field in the spherical coordinate systems can be written as

$$E_\theta^i = \frac{E_0}{k_0 r}\cos\phi\left[j\sum_{n=1}^{N} a_n \hat{J}'_n(k_0 r)\sin\theta\, P'^1_n(\cos\theta) - \frac{1}{\sin\theta}\sum_{n=1}^{N} a_n \hat{J}_n(k_0 r) P^1_n(\cos\theta)\right], \quad (1)$$

$$E_\phi^i = \frac{E_0}{k_0 r}\sin\phi\left[j\frac{1}{\sin\theta}\sum_{n=1}^{N} a_n \hat{J}'_n(k_0 r) P^1_n(\cos\theta) - \sum_{n=1}^{N} a_n \hat{J}_n(k_0 r)\sin\theta\, P'^1_n(\cos\theta)\right], \quad (2)$$

where $a_n = j^{-n}(2n+1)/(n(n+1))$, $\hat{J}_n(k_0 r) = \sqrt{\pi k_0 r/2}\cdot J_{n+1/2}(k_0 r)$, $J_n(\bullet)$ is the n-th order of the Bessel function, $P_n^1(\bullet)$ is n-th order of the Legendre function, N is the truncation mode number, and k_0 is the wave number in free space. The time dependence is $e^{j\omega t}$ and suppressed throughout the paper.

In our PMM, we need to divide the whole physical space into some regions to satisfy the wave equation [1]. For this simple geometry, we divide the whole physical space into two regions, which are outside and inside the dielectric sphere. The electromagnetic fields in these regions are expressed as follows:



(a) Overall view　　(b) Section view

Figure 1. Geometry of a dielectric sphere with a PEC shell.

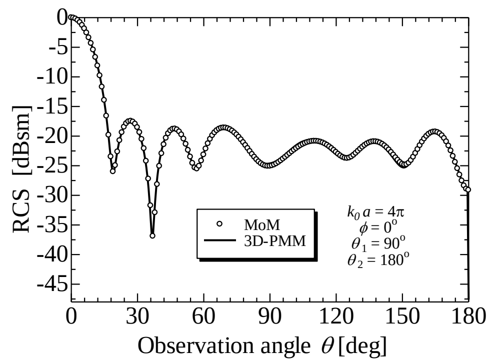


Figure 2. BRCS of the PEC shell when $\theta_1 = 90^\circ$ and $\theta_2 = 180^\circ$.

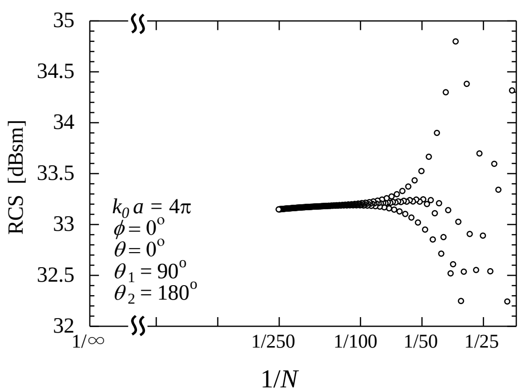


Figure 3. Convergence test of the PEC shell for varying the truncation mode number N.

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Region S_0 : Outside the dielectric sphere

The scattered field can be expressed by using the Hankel function which satisfies the radiation condition. Using a finite sum of modes, it can be approximated as

$$E_\theta^{S_0} = -\frac{E_0}{k_0 r}\cos\phi\left[j\sum_{n=1}^{N} b_n \hat{H}_n^{(2)\prime}(k_0 r)\sin\theta P'^{1}_{n}(\cos\theta) - \frac{1}{\sin\theta}\sum_{n=1}^{N} c_n \hat{H}_n^{(2)}(k_0 r) P_n^1(\cos\theta)\right], \quad (3)$$

$$E_\phi^{S_0} = -\frac{E_0}{k_0 r}\cos\phi\left[j\frac{1}{\sin\theta}\sum_{n=1}^{N} b_n \hat{H}_n^{(2)\prime}(k_0 r) P_n^1(\cos\theta) - \sum_{n=1}^{N} c_n \hat{H}_n^{(2)}(k_0 r)\sin\theta P'^{1}_{n}(\cos\theta)\right], \quad (4)$$

where $\hat{H}_n^{(2)}(k_0 r) = \sqrt{\pi k_0 r/2}\cdot H_{n+1/2}^{(2)}(k_0 r)$, $H_n^{(2)}(\,)$ is the *n*-th order of the second kind of the Hankel function.

Region S_1 : Inside the dielectric sphere

The electromagnetic field in this region can be expressed by using the Bessel function, such as

$$E_\theta^{S_1} = \frac{E_0}{k_1 r}\cos\phi\left[j\sum_{n=1}^{N} d_n \hat{J}'_n(k_1 r)\sin\theta P'^{1}_{n}(\cos\theta) - \frac{1}{\sin\theta}\sum_{n=1}^{N} e_n \hat{J}_n(k_1 r) P_n^1(\cos\theta)\right], \quad (5)$$

$$E_\phi^{S_1} = \frac{E_0}{k_1 r}\sin\phi\left[j\frac{1}{\sin\theta}\sum_{n=1}^{N} d_n \hat{J}'_n(k_1 r) P_n^1(\cos\theta) - \sum_{n=1}^{N} e_n \hat{J}_n(k_1 r)\sin\theta P'^{1}_{n}(\cos\theta)\right], \quad (6)$$

where $k_1 = k_0\sqrt{\mu_r\varepsilon_r}$, μ_r is the relative permeability and ε_r is the relative permittivity.

The unknown coefficients b_n , c_n , d_n , and e_n are determined to satisfy the boundary conditions at sampling points which are placed at the same interval along the θ direction. On the dielectric sphere, the tangential components of the electromagnetic field should satisfy the continuity condition. On the PEC shell, tangential components of the electric field should be vanished.

3. Computational Results

We analyze scattering problems of a PEC shell and investigate computational accuracy. Here, we assume the simple case for $\mu_r = \varepsilon_r = 1$.

Figure 2 shows a comparison of the bistatic radar cross section (BRCS) by 3D-PMM with that by the method of moments (MoM) when $k_0 a = 4\pi$. Here, the PEC shell is placed in the region of $90^\circ \le \theta \le 180^\circ$ and the observation points are at $\phi = 0^\circ$. Dots indicate the BRCS computed by 3D-PMM, the solid line indicates the BRCS by MoM. We can confirm that result of 3D-PMM is almost equivalent to MoM.

Figure 3 shows the convergence test of the PEC shell at the observation point $\theta = 0^\circ$ and $\phi = 0^\circ$ by using 3D-PMM. In this case, we can obtain 3-digit accuracy for 3D-PMM when $N > 103$ is selected.

In Figure 4, we make a comparison of the BRCS by using 3D-PMM with that by MoM when $k_0 a = \pi$. Here, the PEC shell exists in the region of $45^\circ \le \theta \le 180^\circ$ and the observation points are at $\phi = 0^\circ$ .The computational result of 3D-PMM is in a good agreement to that of MoM.

Figure 5 shows the convergence test by using 3D-PMM of the PEC shell at the observation point $\theta = 0^\circ$ and $\phi = 0^\circ$.

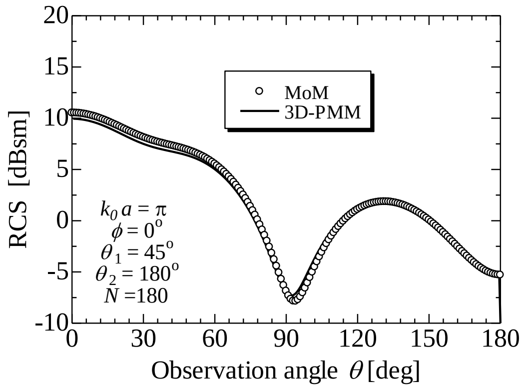

Figure 4. BRCS of the PEC shell when $\theta_1 = 45^\circ$ and $\theta_2 = 180^\circ$.

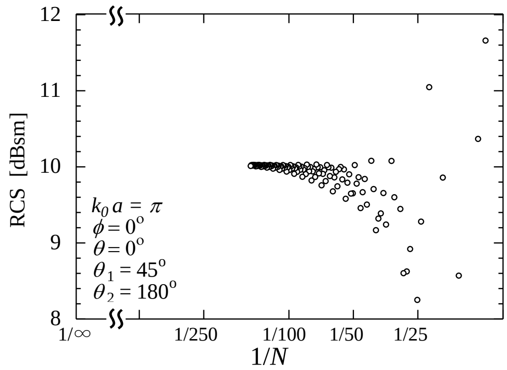

Figure 5. Convergence test of a PEC shell for varying the truncation mode number *N*.

In this case, we can obtain 3-digit accuracy for 3D-PMM when $N > 143$ is selected.

4. Conclusions

In this paper, we discuss electromagnetic scattering from a spherical shell and investigate computational accuracy of our proposed method. Our method can be obtained 3-digit accuracy for 3D-PMM in terms of selecting proper truncation mode numbers.

5. Ackowledgemetns

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6. References

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