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The application of compressive sampling in elastic wave velocity tomography

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Abstract: An innovative elastic wave velocity tomography algorithm based on compressed sampling (CS) is verified in this paper. Conventional elastic wave velocity tomography requires a lot of labor and time. However, the CS algorithm can overcome this defect. The basic principle is to recover the original velocity distribution from random subsampling paths. CS algorithm selects few quantities of paths from a large number of whole measurements. Then, in the visualization stage, the damage location and size can be accurately recovered by L1 minimization optimization algorithm. CS algorithm is more efficient than the conventional algorithms of elastic wave velocity tomography due to its greatly reduced measurement workload and high precision reconstruction.

１．Introduction

An increasing number of aging infrastructure requires non-destructive testing to assess its security situation, as the inside of concrete injure is hard to be organized. Elastic wave velocity tomography is a crucial part of non-destructive testing. Firstly, the sensor receives the elastic wave data, which is emitted by the transmitter in the measurement phase. Secondly is the visualization phase. The velocity distribution could be obtained through iterative techniques for tomographic reconstruction, such as algebraic reconstruction techniques(ART) and simultaneous iterative reconstruction techniques(SIRT). However, these traditional algorithms always cost a huge cost of computation and labor force. The CS algorithm employed in conventional elastic wave velocity tomography could overcome these drawbacks and provide accurate results.

２．Theory of compressive sensing

The CS theory was proposed by D. Donoho, E. Candès and T. Tao et al. [1] in 2006. This theory was proposed for a large amount of data collection and compression, to obtain the data in a compressed form instead of obtaining all the data and then compress it.

The traditional Nyquist sampling theory requires that the sampling frequency must be greater than twice the maximum frequency during sampling in order to preserve the integrity of the original signal. Obviously, if the signal frequency is relatively high, there will be a large number of data. The aspect of compressed sensing needs to be considered the sparsity of the signal rather than the sampling frequency. As long as the signal is sparse, the original signal can be recovered. The primary content of the theory is as follows:

$$y = \Phi\Psi\alpha + n = \Phi x + n = \Theta\alpha + n \qquad (1)$$

where, y is measurement vector, x is the unknown signal, $\alpha \in R^N$, is the vector with K nonzero elements(K<<N, K-sparse), $\Psi \in R^{N\times N}$ is the sparsity basis matrix, transfer matrix Θ has a dimension of m×N, the rows of matrix Θ are much fewer than the columns of Θ, Φ is the measurement matrix, and n is the noise in the process of measurement. Overall, the problem is to recover α from its measurements y, it is an undetermined ill-posed problem. Both signal x and transfer Θ has to meet the restricted isometry property(RIP) condition. Furthermore, several requirements should be satisfied in order to recover x from y. First, every column of matrix Φ must be orthogonal. Second, the cardinality of them should be less than K. Third, the measurement quantity must meet the condition as the following relation: $m > \mu \cdot K \cdot \log(N/K)$, Where μ is a constant with a stationary value. Supposing all requests above are satisfied, in that case, vector x can be precisely reconstructed from vector y. To a broader extent, other random matrices like Gaussian matrix and Bernoulli matrix can also be used as measurement matrix to accomplish this procedure.

Then, a convex optimization algorithm in the following form can be utilized to solve this ill-posed problem.

$$\min \|\alpha\|_1 \text{ subject to } \|\Theta\alpha - y\|_2 < \varepsilon \qquad (2)$$

where ε is the noise boundry.

３．Elastic wave velocity tomography using compressive sensing

The sparsity of concrete destruction is the basis for CS theory to be used in this field. Based on the above statement, velocity distribution could be recovered to determine the damage information. The product can be expressed as follow[2]:

$$y = \Phi\cdot(T - T_0) = \Phi\cdot A\cdot\Delta S \qquad (3)$$

Where T and T_0 are the measured and healthy condition travel time, ΔS is the slowness vector difference between damaged and healthy concrete structure condition. Sparse matrix Φ have some obvious characteristics[3]. Firstly, one and only one entry is equal to unity in each row. Secondly, in each column, nothing but one entry at most is equal to unity. The Bernoulli distribution determines these two properties and the value of another component of matrix Φ will be zero. Then, Eq.(3). could be solved by L1-minimization algorithm, shows as follow:

$$\Delta\hat{S} = \arg\min(\|\Theta\cdot\Delta S - y\|_2 + \lambda\|\Delta S\|_1),\ \Theta = \Phi\cdot A \qquad (4)$$

Where $\Delta\hat{S}$ is reconstructed slowness and λ is the Lagrange multiplier in the ℓ1-minimization algorithm.

４． Simulation result and analysis

In order to verify the CS algorithm is practicable or not, it is essential to compare the CS algorithm with the traditional SIRT algorithm numeral results. The basic information of the model is as follows: the size of the concrete slab is 10*10m. The specimen meshed as 10*10 as well. Twenty sensors are arranged around the whole model. The events are 100 random elastic waves and the signal at the speed of 4000m/s in concrete generally. The original velocity distribution is 4000m/s in healthy condiction and receivers location shown in Figure1.

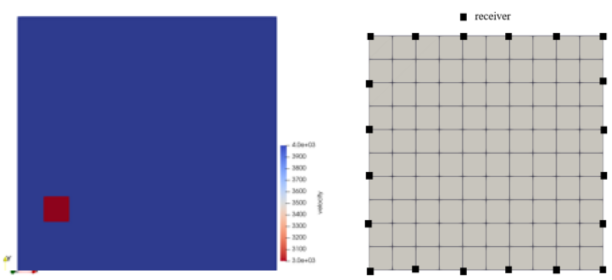
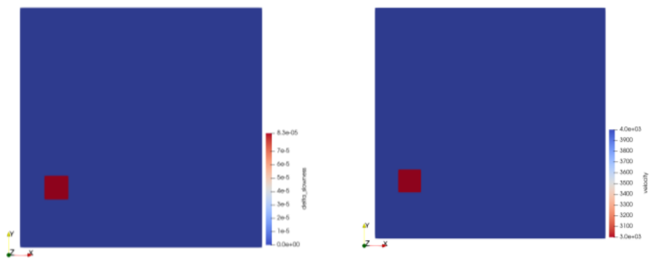


Figure 1. Original velocity distribution and Receivers location

Based on the above ideas and situation, it is crucial to conduct CS theory in a conventional elastic wave velocity model. Due to the destruction characteristics of concrete structures, most of the sample is healthy, and the damaged areas are very sparse. The random matrix Φ adopted 50 measurement paths from 2000 totality. The choice of the number of the paths must satisfy m>u·K·log(N/K), then the size of Φ is 50*2000, and the size of Θ is 50*100. Both meet the RIP and incoherence conditions. Compared with traditional algorithm, CS algorithm choice 50 paths is far less than 2000 original samples. Then, the reconstructive slowness is solved by L1- minimization optimization algorithm.



(a) **(b)**

Figure 2. Damage detection of the simulation model (a) Reconstruction result by CS algorithm. (b) Iteration result by SIRT.

The results in figure2 show that the red part is the damaged objective, and the blue part is the healthy condition, which has healthy velocity distribution. Figure 2.(b) clearly shows that the SIRT iteration result could distinguish the damage target set in advance. Compared with the traditional SIRT algorithm, the CS algorithm could recover the location and size of damage use much fewer measurement data(only 50 paths) than the total(2000 paths).

５． Conclusion

This paper compares the CS algorithm with the traditional algorithm in elastic wave tomography. The results show that the innovative algorithm can accurately restore the localization and range of the concrete damage with a massive decrease of measurement samples which effectively improves detection efficiency.

６． References

[1] Candès EJ, Wakin MB. An introduction to compressive sampling. IEEE Signal Processing Magazine. 2008; 25(2):21±30.

[2] Jiang B, Zhao W, Wang W. Improved Ultrasonic Computerized Tomography Method for STS (Steel Tube Slab) Structure Based on Compressive Sampling Algorithm. Applied Sciences. 2017; 7(5):432.

[3] Wang, Wentao, et al. "The study of compressive sampling in ultrasonic computerized tomography." Structural Health Monitoring and Inspection of Advanced Materials, Aerospace, and Civil Infrastructure 2015. Vol. 9437. International Society for Optics and Photonics, 2015.